Quiz 10 MTH 335 Fall 2026

Monday, October 12, 2026 4:57 PM

Compute $\mathcal{L}\{t^2\}$

Soln: $\mathcal{L}\{t^2\}(s) \overset{\text{def}}{=} \int_0^\infty t^2 e^{-st}\,dt$

$u = t^2 \quad dv = e^{-st}dt$

$du = 2t\,dt \quad v = -\frac{1}{s}e^{-st}$

$$\overset{\text{parts}}{=} -\frac{t^2}{s}e^{-st}\Big|_{t=0}^{t=\infty} + \frac{2}{s}\int_0^\infty te^{-st}\,dt$$

$$= [0 - 0] + \frac{2}{s}\int_0^\infty te^{-st}\,dt$$

as $t\to\infty$, $-\frac{t^2}{s}e^{-st} \to 0$ by l'Hôpital's rule as long as $s>0$

$u = t \quad dv = e^{-st}dt$

$du = dt \quad v = -\frac{1}{s}e^{-st}$

$$\overset{\text{parts again}}{=} -\frac{t}{s}e^{-st}\Big|_{t=0}^{t=\infty} + \frac{2}{s^2}\int_0^\infty e^{-st}\,dt$$

$$= [0-0] + \frac{2}{s^2}\left[-\frac{1}{s}e^{-st}\Big|_{t=0}^{t=\infty}\right.$$

$$= \frac{2}{s^2}\left[0 - \left(-\frac{1}{s}\right)e^0\right]$$

$$= \frac{2}{s^3}$$