Quiz 11 MTH 335 Fall 2026

Monday, October 12, 2026 5:01 PM

Solve

$$\begin{cases} y' + 2y = 4 \\ y(0) = 1 \end{cases}$$

Soln: Take $\mathcal{L}$ to get

$$[s\mathcal{L}\{y\} - \underbrace{y(0)}_{=1 \text{ by given info}}] + 2\mathcal{L}\{y\} = \underbrace{\mathcal{L}\{4\}}_{= \frac{4}{s} \text{ by table}}$$

$$(s+2)\mathcal{L}\{y\} - 1 = \frac{4}{s}$$

$$\mathcal{L}\{y\} = \frac{4}{s(s+2)} + \frac{1}{s+2}$$

But

$$\frac{4}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} \longrightarrow 4 = A(s+2) + Bs$$

$s = -2$: $4 = -2B$, $B = -2$

$s = 0$: $4 = 2A$, $A = 2$

$$= \frac{2}{s} + \frac{(-2)}{s+2}$$

So, we have

$$\mathcal{L}\{y\} = \frac{2}{s} - \frac{2}{s+2} + \frac{1}{s+2}$$

$$= \frac{2}{s} - \frac{1}{s-(-2)}$$

Thus take $\mathcal{L}^{-1}$ of both sides to get

$$y(t) = 2\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s-(-2)}\right\}$$

$$\overset{\text{table}}{=} 2 - e^{-2t}$$