Quiz 9 MTH 335 Fall 2026

Monday, October 12, 2026 4:46 PM

Solve $y'' - 3y' + 2y = \dfrac{e^{3t}}{1+e^t}$ (call this f(t))

Soln: First solve

$$y'' - 3y' + 2y = 0$$

$$\downarrow \text{chr eqt}$$

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0$$

$$\downarrow$$

$$r = 2, 1$$

$$y_h(t) = c_1 e^{2t} + c_2 e^{t}$$

$y_1(t) = e^{2t}$ $\quad$ $y_2(t) = e^{t}$

Now recall

$y_p(t) = u_1 y_1 + u_2 y_2$ is form of our particular soln.

Compute

$$W\{y_1, y_2\}(t) = \det\left(\begin{bmatrix} e^{2t} & e^{t} \\ 2e^{2t} & e^{t} \end{bmatrix}\right) = e^{3t} - 2e^{3t} = -e^{3t}$$

Now we may compute

$$u_1(t) = -\int \frac{y_2 f}{W}$$

$$= -\int \frac{e^t \left[\frac{e^{3t}}{1+e^t}\right]}{-e^{3t}}\, dt$$

$$= \int \frac{e^t}{1+e^t}\, dt$$

$u = 1+e^t$, $du = e^t\, dt$

$$= \int \frac{1}{u}\, du = \ln(u) = \ln(1+e^t)$$

and

$$u_2(t) = \int \frac{y_1 f}{W} = \int \frac{e^{2t}\left[\frac{e^{3t}}{1+e^t}\right]}{-e^{3t}}\, dt$$

$$= -\int \frac{e^{2t}}{1+e^t}\, dt$$

$e^t = u - 1$

$u = 1+e^t$, $du = e^t\, dt$

$$= -\int \frac{e^t}{1+e^t}\, e^t\, dt$$

($e^t = u-1$ in the numerator; $1+e^t = u$ in the denominator; $e^t\, dt = du$)

$$= -\int \frac{u-1}{u}\, du = -\int 1 - \frac{1}{u}\, du$$

$$= -u + \ln(u)$$

$$= -(1+e^t) + \ln(1+e^t)$$

Therefore, the particular soln is

$$y_p(t) = u_1 y_1 + u_2 y_2 = \ln(1+e^t)e^{2t} + \left[-(1+e^t) + \ln(1+e^t)\right]e^t$$

So the general soln is

$$y(t) = y_h(t) + y_p(t)$$

$$= c_1 e^{2t} + c_2 e^t + \ln(1+e^t)e^{2t} + \left[-(1+e^t) + \ln(1+e^t)\right]e^t$$

$-e^t$ $-e^{2t}$

notice this piece can be absorbed here, but doing so isn't strictly necessary!!